Precalculus (Math 097, 107 or 115) Readiness Check

The intent of these exercises is to help you decide whether you are ready for the upgrading course MATH 097, the university transferable precalculus courses (MATH 107 or MATH 115) or whether you should first upgrade your math skills by refreshing with MATH 077 Algebra & Triangle Trigonometry.

All the questions in the following exercises have full solutions. If you find yourself frequently turning to the solutions to help you answer the questions, this is a sign that your background is deficient on this topic. It is important that you be honest with yourself; that is, are you just a bit rusty and the material will come back to you or do you need a comprehensive review. It is unrealistic to think that you can relearn algebra and trigonometry at the same time as you are learning the more advanced concepts and methods in a precalculus course.

- If you struggle with most of the problems or do not remember a large proportion of this material, then you should consider registering in MATH 077 to upgrade your level of mathematical skill.

- If you can work through many of the questions but it takes a while and you struggle with some of them, then likely you are ready for either MATH 097 or MATH 107. These are the easier of the precalculus courses and MATH 107 is designed to prepare you for MATH 108 (Applied Calculus).

- If you can do most of the questions, find some of them easy and struggle with just a few, then you are ready for MATH 115. This is the more challenging precalculus course that will prepare you for MATH 100 (Calculus 1).

If you are feeling uncertain as to how to proceed, then please talk with a math instructor or the Chair of the Mathematics and Statistics Department for help choosing the right course and strategy for you.

A.1 *Evaluate each of the following numerical expressions without using a calculator:*

1. $-\{2-[4-(3-5)]-(5-2)\}$ 2. $-2^4+(-2)^3-2^{-2}$ 3. $\dfrac{1}{3^{-1}-4^{-1}}$

A.2 *Evaluate the following algebraic expressions for the given values of the variables without using a calculator:*

1. a^4-2bc if $a=-2, b=3, c=-4$ 2. $9xy^2$-$3xy^{-2}$ if $x=\dfrac{1}{2},\ y=-\dfrac{1}{3}$

B.1 *Simplify each of the following and write the answers with positive exponents only. Assume all variables are positive real numbers.*

1. $(-3x^5y^{-2}z^0)(8x^3y)$ 2. $\left(\dfrac{a^2b^{-3}c}{2a^{-1}b^{-2}c^3}\right)^{-4}$

B.2 *Simplify each of the following exponential expressions. Assume that all variables are positive real numbers and write your answers as: a) positive exponent, and b) in radical notation.*

1. $\dfrac{x^{1/3}x^{-5/3}}{x^{-2/3}}$ 2. $\left(x^{1/3}y^{3/2}\right)\left(x^{-1/2}y^{1/4}\right)^2$

C.1 *Simplify each of the following radical expressions. Assume that all variables are positive real numbers.*

1. $\sqrt{18}-\sqrt{50}-3\sqrt{12}-2\sqrt{75}$ 2. $-\sqrt{75x^3y^7}$ 3. $(2+3\sqrt{5})(3-7\sqrt{10})$

C.2 *Simplify the following radical expression by rationalizing the denominator.*

1. $\dfrac{2}{5-\sqrt{3}}$ 2. $\dfrac{\sqrt{5}-\sqrt{7}}{2\sqrt{5}+\sqrt{7}}$

D.1 *Simplify each of the following polynomial expressions.*

1. $3[3-2(x-2)-2(3-4x)]$ 2. $-4(2x-3)^2-(5x-1)(2x+3)$

3. $(3x-4)(7x^2+5x-1)$

D.2 *Factor completely each of the following polynomial expressions.*

1. $100x^2-81$ 2. $x^2-7x+12$ 3. $6x^2+26x-20$

4. $x^2y^2+ab-ay^2-bx^2$

D.3 *Simplify: perform each of the following operations.*

1. $\dfrac{2x^2-8}{x^2-4x+4}$ 2. $\dfrac{x^2-5x-6}{x^2-6x}\cdot\dfrac{6x}{12x+12}$

3. $\dfrac{2}{x^2-3x+2}+\dfrac{6}{x^2-1}$

D4. *Perform the indicated divisions.*

1. $\dfrac{21x^3-35x^2+14x-7}{7x}$ 2. $(3x^3-5x^2+10x-3)\div(3x+1)$ Long division.

E.1 *Solve the linear equations*

1. $\frac{5}{2}x-7=18$
2. $\frac{7}{5}(x-1)-\frac{3}{2}(x-2)=1$
3. $6w-5d+7h=80$ for w
4. $a=\frac{3}{b}(b-y)$ for "b"

E.2 *Solve the quadratic equations*

1. $2x^2=14x$
2. $3x^2=5x-1$
3. $3x^2-4x-1=0$, by completing the square.

E.3 *Solve the following linear inequalities and give your answer in interval notation*

1. $8-3x\le 2$
2. $-\frac{1}{2}\le\frac{3-2x}{2}<\frac{5}{4}$

E.4 *Solve the rational equations*

1. $\frac{8}{y}-\frac{1}{3}=\frac{5}{y}$
2. $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^2-6x+8}$

E.5 *Solve the radical equations*

1. $\sqrt{2x+5}=7$
2. $\sqrt{3x+1}-\sqrt{x+4}=1$

F.1 *Find the equation of the line that satisfies each of the following conditions.*

1. Passes through the points $(-2,3)$ and $(5,-6)$. Write your answer in standard form.

2. Perpendicular to line whose equation is $y=\frac{3}{4}x-5$, and contains the point (-3, 1).

3. Parallel to the line whose equation is $-2x+5y=12$, and passes through the point (10, -1).

F.2 *Graph each of the following functions. Give the coordinates of all intercepts and any other important points.*

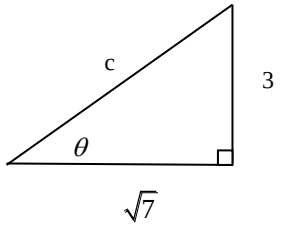
1. $2x+3y=12$
2. $f(x)=-2(x+3)^2+4$

G *Solve the system of equations.*

1. $\begin{aligned}4x+3y&=4\\6x-5y&=-32\end{aligned}$

H.1 *Find the indicated trigonometric ratios for the following right triangle.*

1. Find: $\tan\theta$, and $\csc\theta$



H.2 *Use your calculator to evaluate the following trigonometric expressions to three decimal places:*

1. $\sin 24.5^0$
2. $\sec\left(121.7^0\right)$

H.3 Solving Right Triangles:

If the hypotenuse of a right triangle has a length of 15 and one of the angles is 27 degrees then find the lengths of the remaining two sides and the size of the measure of the other angle.

SOLUTIONS

A.1

1. $-\{2-[4-(3-5)]-(5-2)\} = -\{2-[6]-(3)\}$

$= -\{-7\}$

$= \boxed{7}$

2. $-2^4 + (-2)^3 - 2^{-2} = -16 - 8 - \frac{1}{2^2}$

$= -24 - \frac{1}{4}$

$= \frac{-96}{4} - \frac{1}{4}$

$= \boxed{\frac{-97}{4} \text{ or } -24\,1/4}$

3. $\frac{1}{3^{-1} - 4^{-1}} = \frac{1}{\frac{1}{3} - \frac{1}{4}} = \frac{1}{\frac{1}{12}} = \boxed{12}$

A.2

1. $a^4 - 2bc = (-2)^4 - 2(3)(-4)$

$= 16 + 24$

$= \boxed{40}$

2. $9xy^2 - 3xy^{-2}$ if $x = \frac{1}{2}, y = -\frac{1}{3}$

$= 9\left(\frac{1}{2}\right)\left(\frac{-1}{3}\right)^2 - 3\left(\frac{1}{2}\right)\left(\frac{-1}{3}\right)^{-2}$

$= 9\left(\frac{1}{2}\right)\left(\frac{1}{9}\right) - 3\left(\frac{1}{2}\right)\left(\frac{-3}{1}\right)^2$

$= \frac{9}{18} - \frac{3}{2}\left(\frac{9}{1}\right)$

$= \frac{1}{2} - \frac{27}{2}$

$= \frac{-26}{2} = \boxed{-13}$

B.1

1. $\left(-3x^5y^{-2}z^0\right)\left(8x^3y\right)$

$= -24x^8y^{-1}$

$= \boxed{\frac{-24x^8}{y}}$

(Recall $z^0 = 1$)

2. $\left(\frac{a^2b^{-3}c}{2a^{-1}b^{-2}c^3}\right)^{-4} = \left(\frac{a^3}{2^1b^1c^2}\right)^{-4}$

$= \frac{a^{-12}}{2^{-4}b^{-4}c^{-8}} = \boxed{\frac{16b^4c^8}{a^{12}}}$

B.2

1. $\frac{x^{1/3}x^{-5/3}}{x^{-2/3}} = x^{1/3 - 5/3 - (-2/3)}$

$= x^{-2/3}$

$= \boxed{\frac{1}{x^{2/3}} = \frac{1}{\sqrt[3]{x^2}}}$

2. $\left(x^{1/3}y^{3/2}\right)\left(x^{-1/2}y^{1/4}\right)^2 = \left(x^{1/3}y^{3/2}\right)\left(x^{-1}y^{1/2}\right)$

$= x^{\frac{1}{3} - \frac{3}{3}}y^{\frac{3}{2} + \frac{1}{2}}$

$= x^{-2/3}y^{4/2}$

$= \boxed{\frac{y^2}{x^{2/3}} = \frac{y^2}{\sqrt[3]{x^2}}}$

C.1

1.
$$\begin{aligned}&\sqrt{18}-\sqrt{50}-3\sqrt{12}-2\sqrt{75}\\&=3\sqrt{2}-5\sqrt{2}-6\sqrt{3}-10\sqrt{3}\\&=\boxed{-2\sqrt{2}-16\sqrt{3}}\end{aligned}$$

2.
$$\begin{aligned}&-\sqrt{75x^3y^7}=-\sqrt{25\cdot 3\cdot x^2\cdot x\cdot y^6\cdot y}\\&=\boxed{-5xy^3\sqrt{3xy}}\end{aligned}$$

3.
$$\begin{aligned}&\left(2+3\sqrt{5}\right)\left(3-7\sqrt{10}\right)\\&=6-14\sqrt{10}+9\sqrt{5}-21\sqrt{50}\\&=\boxed{6-14\sqrt{10}+9\sqrt{5}-105\sqrt{2}}\end{aligned}$$

C.2

1.
$$\begin{aligned}&\left(\frac{2}{5-\sqrt{3}}\right)\cdot\left(\frac{5+\sqrt{3}}{5+\sqrt{3}}\right)=\frac{10+2\sqrt{3}}{25-3}\\&=\frac{10+2\sqrt{3}}{22}\\&=\boxed{\frac{5+\sqrt{3}}{11}}\end{aligned}$$

2.
$$\begin{aligned}&\left(\frac{\sqrt{5}-\sqrt{7}}{2\sqrt{5}+\sqrt{7}}\right)\left(\frac{2\sqrt{5}-\sqrt{7}}{2\sqrt{5}-\sqrt{7}}\right)=\frac{10-\sqrt{35}-2\sqrt{35}+7}{20-7}\\&=\boxed{\frac{17-3\sqrt{35}}{13}}\end{aligned}$$

D.1

1.
$$\begin{aligned}&3\left[3-2\left(x-2\right)-2\left(3-4x\right)\right]\\&=3\left[3-2x+4-6+8x\right]\\&=3\left[1+6x\right]\\&=\boxed{18x+3}\end{aligned}$$

2.
$$\begin{aligned}&-4\left(2x-3\right)^2-\left(5x-1\right)\left(2x+3\right)\\&=-4\left(4x^2-12x+9\right)-\left(10x^2+13x-3\right)\\&=-16x^2+48x-36-10x^2-13x+3\\&=\boxed{-26x^2+35x-33}\end{aligned}$$

3.
$$\begin{aligned}&\left(3x-4\right)\left(7x^2+5x-1\right)=21x^3+15x^2-3x-28x^2-20x+4\\&=\boxed{21x^3-13x^2-23x+4}\end{aligned}$$

D.2

1. $100x^2-81=\boxed{\left(10x-9\right)\left(10x+9\right)}$

2. $x^2-7x+12=(x-3)(x-4)$

3.
$$\begin{aligned}&6x^2+26x-20=2\left(3x^2+13x-10\right)\\&=2\left(\frac{3x+15}{3}\right)\left(\frac{3x-2}{1}\right)\\&=\boxed{2\left(x+5\right)\left(3x-2\right)}\end{aligned}$$

4.
$$\begin{aligned}&x^2y^2+ab-ay^2-bx^2\\&=x^2y^2-ay^2-bx^2+ab\\&=y^2\left(x^2-a\right)-b\left(x^2-a\right)\\&=\boxed{\left(x^2-a\right)\left(y^2-b\right)}\end{aligned}$$

D.3

1.
$$\begin{aligned}&\frac{2x^2-8}{x^2-4x+4}=\frac{2\left(x^2-4\right)}{\left(x-2\right)^2}\\&=\frac{2\left(x-2\right)\left(x+2\right)}{\left(x-2\right)^2}\\&=\boxed{\frac{2\left(x+2\right)}{x-2}},\ x\neq\pm 2\end{aligned}$$

2.
$$\begin{aligned}&\frac{x^2-5x-6}{x^2-6x}\cdot\frac{6}{12x+12}=\frac{\left(x-6\right)\left(x+1\right)}{x\left(x-6\right)}\cdot\frac{6}{12(x+1)}\\&=\boxed{\frac{1}{2x}}\end{aligned}$$

3.

$$\frac{2}{x^2-3x+2}+\frac{6}{x^2-1}$$
$$=\frac{2}{(x-2)(x-1)}+\frac{6}{(x-1)(x+1)}$$
$$=\frac{2(x+1)+6(x-2)}{(x-2)(x-1)(x+1)}$$
$$=\boxed{\frac{8x-10}{(x-2)(x-1)(x+1)}}$$

D.4

1.
$$\frac{21x^3-35x^2+14x-7}{7x}$$
$$=\frac{21x^3}{7x}-\frac{35x^2}{7x}+\frac{14x}{7x}-\frac{7}{7x}$$
$$=\boxed{3x^2-5x+2-\frac{1}{x}}$$

2.
$$3x+1\overline{)\,3x^3-5x^2+10x-3}$$
$$x^2-2x+4$$
$$-\underline{(3x^3+x^2)}$$
$$-6x^2+10x$$
$$-\underline{(-6x^2-2x)}$$
$$12x-3$$
$$-\underline{(12x+4)}$$
$$-7$$

$Q(x)=x^2-2x+4$

$R(x)=-7$

E.1

1.
$$\frac{5}{2}x-7=18$$
$$5x-14=36$$
$$5x=50$$
$$\boxed{x=10}$$

2.
$$\frac{7}{5}(x-1)-\frac{3}{2}(x-2)=1 \text{ LCD } =10$$
$$\left[\frac{7}{5}(x-1)-\frac{3}{2}(x-2)=1\right]\cdot\frac{10}{1}$$
$$14(x-1)-15(x-2)=10$$
$$14x-14-15x+30=10$$
$$-x=-6$$
$$\boxed{x=\{6\}}$$

3. $6w-5d+7h=80$ for w
$$6w=5d-7h+80$$
$$w=\boxed{\frac{5d-7h+80}{6}}$$

4.
$$a=\frac{3}{b}(b-y) \text{ for "}b\text{"}$$
$$ab=3b-3y$$
$$3y=3b-ab$$
$$3y=b(3-a)$$
$$\boxed{b=\frac{3y}{3-a}}$$

E.2

1. $2x^2 = 14x$

$2x^2 - 14x = 0$

Factor: $2x(x-7) = 0$

$\boxed{x = \{0, 7\}}$

2. $3x^2 = 5x - 1 \Rightarrow 3x^2 - 5x + 1 = 0$

Can't factor, use quad formula:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a = 3, b = -5, c = 1$

$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3}$

$\boxed{x = \dfrac{5 \pm \sqrt{13}}{6} \sim 1.434, 0.232}$

3. $3x^2 - 4x - 1 = 0$ make coeff. 1

$\dfrac{3x^2 - 4x - 1 = 0}{3} \Rightarrow x^2 - \dfrac{4}{3}x = \dfrac{1}{3}$

$\dfrac{1}{2}\left(\dfrac{4}{3}\right) = \dfrac{2}{3} \Rightarrow \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

$x^2 - \dfrac{4}{3}x + \boxed{\dfrac{4}{9}} = \dfrac{1}{3} + \boxed{\dfrac{4}{9}}$

$\left(x - \dfrac{2}{3}\right)^2 = \dfrac{7}{9}$

$x - \dfrac{2}{3} = \pm\dfrac{\sqrt{7}}{3} \Rightarrow \boxed{x = \dfrac{2 \pm \sqrt{7}}{3}}$

E.3

1. $8 - 3x \le 2$

$-3x \le -6$

$\boxed{x \ge 2 \;\Rightarrow\; [2, \infty)}$

2. $\left[-\dfrac{1}{2} \le \dfrac{3 - 2x}{2} < \dfrac{5}{4}\right] \cdot \dfrac{4}{1}$

$-2 \le 2(3 - 2x) < 5$

$-2 \le 6 - 4x < 5$

$-8 \le -4x < -1$

$\dfrac{-8 \ge -4x > -1}{-4}$

$\boxed{2 \ge x > \dfrac{1}{4} \;\Rightarrow \left(\dfrac{1}{4}, 2\right]}$

E.4

1. $\dfrac{8}{y} - \dfrac{1}{3} = \dfrac{5}{y}$ LCD $= 3y, y \ne 0$

$\left[\dfrac{8}{y} - \dfrac{1}{3} = \dfrac{5}{y}\right] \cdot \dfrac{3y}{1}$

$24 - y = 15$

$\boxed{y = \{9\}}$ Checks

2. $\dfrac{x}{x-2} + \dfrac{1}{x-4} = \dfrac{2}{x^2 - 6x + 8} \Rightarrow \dfrac{x}{x-2} + \dfrac{1}{x-4} = \dfrac{2}{(x-4)(x-2)}$

LCD$=(x+4)(x+2), \;\; x \ne 2, 4$

$\left[\dfrac{x}{x-2} + \dfrac{1}{x-4} = \dfrac{2}{(x-4)(x-2)}\right] \cdot \dfrac{(x-4)(x-2)}{1}$

$x(x-4) + (x-2) = 2 \Rightarrow x^2 - 4x + x - 2 = 2$

$x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1) = 0$

$x = \{-1, 4\}$ Note: 4 is rejected $\boxed{\therefore \; x = \{-1\}}$

E.5

1. $(\sqrt{2x+5})^2 = 7^2$
$2x+5=49$
$2x=44$
$\boxed{x=22}$

2. $\sqrt{3x+1}-\sqrt{x+4}=1$
$(\sqrt{3x+1})^2=(1+\sqrt{x+4})^2$
$3x+1=1+2\sqrt{x+4}+x+4$
$(2x-4)^2=\left(2\sqrt{x+4}\right)^2$
$4x^2-16x+16=4(x+4)$
$4x^2-16x+16=4x+16$
$4x^2-20x=0$
$4x(x-5)=0$
$x=0 \quad \text{or} \quad x=5$
Check: $\sqrt{3(0)+1}-\sqrt{(0)+4}\neq 1$
$\sqrt{3(5)+1}-\sqrt{(5)+4}=1$
So, $x=5$, since $x=0$ does not check

F.1

$m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-6-3}{5-(-2)}=\boxed{-\dfrac{9}{7}}$

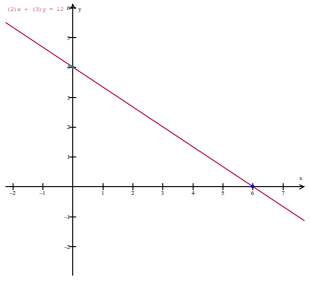
1. using point-slope form:
$y-y_1=m(x-x_1)\Rightarrow y-3=-\dfrac{9}{7}(x+2)$
$7y-21=-9x-18\Rightarrow \boxed{9x+7y=3}$

2. $m=\dfrac{3}{4}$, since $\perp$, $m=-\dfrac{4}{3},(-3,1)$
$y=mx+b\Rightarrow 1=\left(-\dfrac{4}{3}\right)(-3)+b$
$b=-3, \quad \Rightarrow \quad \therefore \quad \boxed{y=-\dfrac{4}{3}x-3}$

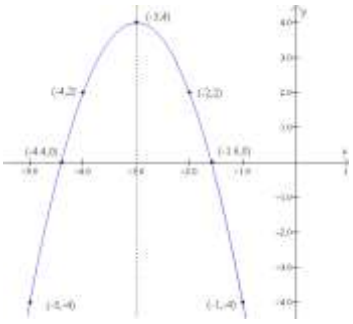
3. $-2x+5y=12$
$5y=2x-12$
$y=\dfrac{2}{5}x-\dfrac{12}{5} \quad m=\dfrac{2}{5}$, since $//$, $m=\dfrac{2}{5},(10,-1)$
$y=mx+b\Rightarrow -1=\left(\dfrac{2}{5}\right)(10)+b$
$b=-5, \quad \Rightarrow \quad \therefore \quad \boxed{y=\dfrac{2}{5}x-5}$

F.2

1. $2x+3y=12$
$3y=-2x+12$
$y=-\dfrac{2}{3}x+4,$
$m=-\dfrac{2}{3}, b=4$



For x int, let y=0 : $2x+3(0)=12\Rightarrow x=6 \quad (6,0)$
For yint (see above) or let x=0 $\quad 2(0)+3y=12\Rightarrow y=4 \quad (0,4)$



$f(x) = -2(x+3)^2 + 4, V@(-3,4)$, opens down,

intercepts:

for yint, let x=0 $\Rightarrow$ f(0)= $-2(0+3)^2 + 4 = -14$

for x int, let y=0 $\Rightarrow 0 = -2(x+3)^2 + 4$

2. $-4 = -2(x+3)^2$

$2 = (x+3)^2$

$\pm\sqrt{2} = x+3$

$x = -3 \pm \sqrt{2}$

$x \approx -1.6, -4.4$

intercepts : $(-1.6, 0)\ (-4.4, 0)(0, -14)$

G

$3(4x+3y=4) \Rightarrow 12x+9y=12$

$-2(6x-5y=-32) \Rightarrow -12x+10y=64$

$19y = 76$

$y = 4$

1. resub into eqn 1. $4x+3(4)=4$

$4x = -8$

$x = -2$

$(-2, 4)$

H.1

1. Use Pythagoras to find "a"

$c^2 = 3^2 + \left(\sqrt{7}\right)^2 \Rightarrow c^2 = 16, \therefore c = 4.$

$\boxed{\tan\theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}}$, $\boxed{\csc\theta = \frac{4}{3}}$

H.2

1. $\sin 24.5° = 0.415$

2. $\sec 121.7° = \frac{1}{\cos 121.7°} = -1.903$

H.3

A + B + C =180, so if A is the missing angle then A + 27 + 90 = 180 so A = 63 degrees.

If *a* is the length of the side opposite the $27°$ angle then $\sin 27° = \frac{a}{15} \Rightarrow a = 15\sin 27° = 6.81$

If *b* is the length of the remaining side then $\cos 27° = \frac{b}{15} \Rightarrow b = 15\cos 27° = 13.37$