



## Am I Ready for MATH 100?

Calculus will likely be one of the most challenging courses you will take in your first year university program. In order to be successful:

- You must have a strong and recent algebra and trigonometry background.
- You need to have a clear and realistic idea about the commitment of time and energy necessary.

The intent of this booklet is to help you decide whether you are ready for Math 100 or whether you should first refresh in a precalculus course, Math 105 or 115. (Note that these precalculus courses do carry University Transfer Credit and can be used for student loan purposes!).

You should recognize and be able to do most of the exercises presented here.

- If you have not seen or do not remember a large proportion of this material, you are not ready for Calculus and will want to register in one of the precalculus courses, Math 105 or Math 115
- If you can work through most of the questions but it takes a long time and you struggle with many of the questions, then you have a decision to make. You can stay in Math 100 knowing that you will need to devote more time to doing math homework and will likely settle for a lower final grade or you can transfer to a precalculus course to improve your basic math skills and then register in Math 100 next semester.

This booklet is intended as a self-diagnostic tool and is not as comprehensive as the Principles of Math 12 or a precalculus course.

The format of this booklet is more like a workbook rather than a textbook. The odd numbered questions have worked out solutions and the subsequent even numbered question is similar and has only an answer. If you find yourself frequently turning to the solutions/answers to help you answer the questions, this is a strong sign that your background is deficient on this topic. It is important that you be honest with yourself at this point. That is, are you just a bit rusty and the material will come back to you or do you need a comprehensive review. It is unrealistic to think that you can relearn algebra and trigonometry at the same time that you are learning new concepts and ideas in Calculus.

By the end of the first week of classes you should have completed or attempted all of the questions and be in a good position to decide whether to stay in Math 100 or transfer to an upgrading course. Please talk with your instructor or the Chair of the Mathematics Department to help you choose the right course and strategy for you.

## Section A: Order of Operations

### A.1 Evaluating Numerical Expressions

Evaluate the following expressions for the given values of the variables without using a calculator:

- $\frac{-12x^2 - y^3}{3xy^2 - 4y}$  if  $x = -2$   $y = -3$
- $\frac{-3x^2 - 2xy^3}{3x^2y^2 - 2y}$  if  $x = -\frac{2}{3}$   $y = \frac{1}{2}$
- $3x^2(2x+15)^{-\frac{1}{2}} - x^3(2x+15)^{-\frac{3}{2}}$  if  $x = -3$
- $6x(8-x^2)^{\frac{4}{3}} - 8x^2(8-x^2)^{\frac{1}{3}}$  if  $x = -4$
- $(x^3 - y^{-2})^{-1}$  if  $x = \frac{1}{2}$   $y = -3$
- $\frac{x^{-1} + y^{-1}}{x^{-1} - y^{-2}}$  if  $x = 4$   $y = 2$

### A.2 Simplifying Algebraic Expressions

Simplify the following expressions by using an appropriate method. Give your answer using only positive exponents.

- $(3x^2 - 2)(x+1) - 6 - 2(x-2)^3$
- $5 - 3(2x+3)^2 - (3x+1)(x-4)$
- $\sqrt{(x^2 - 1)^2 + 4x^2}$
- $\sqrt{x^2 + 4} + \sqrt{9x^2 + 36}$
- $\frac{1}{x+3+h} - \frac{1}{x+3}$
- $\frac{1}{x} - \frac{1}{x-4}$
- $2(x+2)^{-1} - 2x(x+2)^{-2}$
- $(x^{-1} - 2^{-1})^{-2}$

## **Section B: Factoring and its Applications**

### **B.1 Basic Techniques**

**Factor out the greatest common factor and simplify:**

1.  $6x^5(4x^2 - 1)^5 + 40x^7(4x^2 - 1)^4$

2.  $6x(x^2 + 1)^2(5x + 1)^2 + 10(5x + 1)(x^2 + 1)^3$

**Factor completely:**

3.  $x^4 - 13x^2 - 48$

4.  $2x^2 + 7x - 15$

**Factor by grouping:**

5.  $2x^3 - 5x^2 - 18x + 45$

6.  $4x^3 + 2x^2y - 4xy^2 - 2y^3$

### **B.2 Simplifying Expressions**

**Reduce the following expressions to lowest terms:**

1.  $\frac{4x^2 - 8x - 12}{2x^2 - 5x - 3}$

2.  $\frac{3x^4 - 48}{x^2 + 4x - 12}$

**Use an appropriate technique to simplify the following expressions:**

3.  $\frac{\frac{4}{x^2} - \frac{4}{9}}{x - 3}$

4.  $\frac{\frac{1}{x^3} - \frac{1}{8}}{x - 2}$

**Find the lowest common denominator and combine the following terms:**

5.  $\frac{4}{x^2 - 1} + \frac{x}{x^2 - 2x - 3}$

6.  $\frac{2x}{(x - 1)^2} - \frac{4}{x^2 - x}$

### **B.3 Solving Equations**

**Use factoring and other appropriate techniques to find all real number solutions to the following equations:**

1.  $2x^2 - x - 15 = 0$

2.  $x^4 - 5x^2 - 36 = 0$

3.  $3x^2 - 2x - 4 = 0$

4.  $2x^2 - 3x + 5 = 0$

5.  $\frac{1}{x} - \frac{2x}{x+3} = 0$

6.  $\frac{2x}{x-2} - \frac{x^2-3}{(x-2)^2} = 0$

7.  $\frac{x+1}{\sqrt{x^2+2x+13}} = \frac{1}{2}$

8.  $\sqrt{x^2+1} + \frac{x^2-3x}{\sqrt{x^2+1}} = 0$

9.  $3(x+4)^2(2x-1)^{5/2} + 5(x+4)^3(2x-1)^{3/2} = 0$  (Hint: Factor out the GCF.)

10.  $6x(x^2-1)^2(x+3)^4 + 4(x+3)^3(x^2-1)^3 = 0$  (Hint: Factor out the GCF.)

**Solve the following equations for the indicated variable:**

11.  $x = \frac{5-y}{y+4}$  for  $y$

12.  $A = 2lw + 2lh + 2hw$  for  $w$

### **B.4 Solving Inequalities**

**Solve the following inequalities and give your answer using interval notation:**

1.  $x - 5\left(\frac{1}{2}x - \frac{2}{3}\right) - 1 \geq 0$

2.  $3 - 5(2x + 3) + 7x < 0$

3.  $x^2 - 5x - 14 > 0$

4.  $\frac{3-x}{x+1} \geq 0$

## **Section C: Analytic Geometry and Basic Graphing**

### **C.1 The Distance, Midpoint, Slope and Point-Slope Formulas**

Given the following pairs of points find:

- The distance between the points.
- The midpoint of the line segment joining the points.
- The slope of the line going through the points.
- The slope of any line perpendicular to the line going through the points.
- The equation of the line passing through the points.

1.  $(-2, -1), (6, -3)$

2.  $(3, -4), (-2, 1)$

Find the  $x$  and  $y$  intercepts and then use these to sketch the graphs of:

3.  $y = \frac{5}{3}x - \frac{1}{2}$

4.  $3x + 2y - 5 = 0$

### **C.2 Quadratic, Rational and Radical Functions**

For the following parabolas:

- Find the  $x$  and  $y$  intercepts.
- Complete the square to find the vertex.
- Sketch the graph.

1.  $y = x^2 - 6x + 5$

2.  $y = -2x^2 - 16x - 24$

For the following rational functions:

- Find the  $x$  and  $y$  intercepts.
- Identify the vertical and horizontal asymptotes.
- Sketch the graph.

3.  $y = \frac{2}{x-3} - 1$

4.  $y = \frac{2x-1}{x+1}$

For the following radical functions:

- Identify the domain.
- Find the  $x$  and  $y$  intercepts.
- Sketch the graph.

5.  $y = -\sqrt{x+3}$

6.  $y = \sqrt{1-x} - 2$

## **Section D: Functions and Function Notation**

### **D.1 Domain and Range**

Find the domain and range of the following functions:

1.  $f(x) = \sqrt{2x+1} - 2$

2.  $f(x) = 5 - \sqrt{3-2x}$

3.  $f(x) = \frac{3}{5x-4} + 1$

4.  $f(x) = \frac{3x-1}{x+2}$

5.  $f(x) = \sqrt[3]{x-2} + 1$

6.  $f(x) = \frac{1}{\sqrt[3]{x^2-4}}$

### **D.2 Using Function Notation and Evaluating Difference Quotients**

Evaluate the following for the given function:

1.  $f(-\frac{2}{3})$  ,  $f(x) = \frac{-x^2 + 2x + 2}{4x - 3}$

2.  $f(-\frac{3}{4})$  ,  $f(x) = \frac{5 - 2\sqrt{x^2 + 1}}{x}$

3.  $f(3a)$  ,  $f(x) = \sqrt{x^2 + 9} + \frac{1}{\sqrt{x^2 + 9}}$

4.  $f(a-3)$  ,  $f(x) = x^2 - 2x + 5$

For each of the functions below, evaluate and simplify the difference quotient:

$$\frac{f(x+h) - f(x)}{h}$$

5.  $f(x) = -2x^2 + 3x - 1$

6.  $f(x) = x^3 + 5$

7.  $f(x) = \frac{1}{3x+2}$

8.  $f(x) = \frac{2}{4-x}$

9.  $f(x) = \sqrt{x-1}$  (Rationalize the numerator)

10.  $f(x) = \sqrt{1-2x}$

### D.3 Multiplication, Composition and Decomposition of Functions

For each pair of functions, evaluate:  $f(x)g(x)$  ,  $f(g(x))$  and  $g(f(x))$  :

1.  $f(x) = 2x + 5$  ,  $g(x) = x^3 + 1$
2.  $f(x) = 2x^2 + 5$  ,  $g(x) = \sqrt{2x - 1}$
3.  $f(x) = x^3 + 8$  ,  $g(x) = \frac{1}{\sqrt[3]{x}}$
4.  $f(x) = \frac{2}{3x - 4}$  ,  $g(x) = 8 - 6x$

Find functions  $f(x)$  and  $g(x)$  such that  $k(x) = f(g(x))$  for the following:  
(There are different correct answers for each question.)

5.  $k(x) = (4x^2 - 2x - 1)^5$
6.  $k(x) = \sqrt{9x^2 + 25}$
7.  $k(x) = (3x + 2)^4 + \frac{1}{(3x + 2)^2}$
8.  $k(x) = \frac{1}{\sqrt[5]{(x^3 - 27)^2}}$

### D.4 Constructing Functions for Use in Applications

Construct a function as needed for each of the following problems:

1. A student has test scores of 82, 75 and 89. If their score on the fourth test is  $x$ , then write their average  $A$  on all four tests as a function of  $x$ .
2. A piece of wire is 50 cm long and is cut into two pieces. The pieces are then used to form a square and a circle. If the length of the piece used to form the circle has a length of  $x$  cm, then write the total combined area  $A$  of the circle and the square as a function of  $x$ .
3. The perimeter of a rectangle is known to be 45 meters. Write the area  $A$  of this rectangle as a function of only the width  $w$  of the rectangle.
4. The surface area of a cylinder is given by  $S = 2\pi r h + 2\pi r^2$  and the volume is given by  $V = \pi r^2 h$ . If the volume of a cylinder is known to be 500 cc , then write the surface area as a function of only the radius  $r$  of the cylinder.
5. A point lies on the curve  $y = x^2 + 1$  . Write the distance  $D$  of this point from the point  $( 2, 3 )$  as a function of  $x$  .
6. A triangle is formed in Quadrant I using the  $x$  and  $y$  axes and the line  $2x + 3y - 6 = 0$  . A rectangle is drawn inside this triangle so that two sides lie on the axes and the top right corner of the rectangle is on the line. Write the area  $A$  of the rectangle as a function of the  $x$  coordinate of the point in the top right corner.

## Section E: Trigonometry

### E.1 Basic Computations

Convert the following degree measurement to radians or radian measurement to degrees:

1.  $135^\circ$                       2.  $\frac{7\pi}{6}$                       3. 1.15                      4.  $3.42^\circ$

Evaluate the following. You do not need to use a calculator for questions 5 and 6.

5.  $\tan 30^\circ$                       6.  $\sin \frac{5\pi}{3}$                       7.  $\cos 5.8$                       8.  $\sec \frac{3\pi}{5}$

Find the acute angle  $\theta$  that satisfies each of the following equations. Give your answer in both radians and degrees. You do not need to use a calculator for questions 9 and 10.

9.  $\cos \theta = \frac{1}{2}$                       10.  $\sin \theta = \frac{\sqrt{2}}{2}$                       11.  $\cot \theta = 2.86$                       12.  $\csc \theta = 1.25$

Find all angles  $\theta$  that satisfy each of the following equations. Give your answers in radians. You do not need to use a calculator for questions 13, 14 and 15.

13.  $\cos \theta = \frac{1}{2}$                       14.  $\sin \theta = \frac{\sqrt{2}}{2}$                       15.  $\tan \theta = -\frac{1}{\sqrt{3}}$                       16.  $\sin \theta = -.6$

## E.2 Algebraic Computations and using Trig Identities

**You should know the following definitions and basic trig identities:**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

**Note that some of the above identities are interrelated, for example:**

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \Rightarrow \tan^2 \theta + 1 = \sec^2 \theta$$

$$\sin 2x = \sin(x + x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x$$

**Simplify the following expressions using appropriate algebraic techniques and/or trig identities:**

1.  $\frac{1 - \cos^2 x}{1 - \sin^2 x}$

2.  $\frac{\sec^2 x - \tan^2 x}{\sec^2 x - 1}$

3.  $\frac{\cos^2 x}{1 - \sin x}$

4.  $\frac{1 - \sec x}{\tan^2 x}$

5.  $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$

6.  $\frac{1}{\tan x - \sec x} - \frac{1}{\tan x + \sec x}$

7.  $\sin(x + y) - \cos(x - y)$  , Use factoring by grouping to get your final answer.

8.  $\sin(x + y)\sin(x - y)$

9.  $2 \sin 3x \cos 3x$

10.  $\cos^2 4x - \sin^2 4x$

11.  $\frac{\sin 2x - 2 \sin^2 x}{\cos 2x}$

12.  $\frac{\sin 2x}{1 + \cos 2x}$

### E.3 Using Trig Identities When Solving Trig Equations

Use a trig identity to help in solving the following equations. Give your answer in radians and find all solutions on the interval  $[0, 2\pi)$ :

1.  $\sin 2x + \sin x = 0$

2.  $\sin 2x + 2\cos x = 0$

3.  $\cos 2x + \sin x = 0$

4.  $\cos 2x = -\cos x$

5.  $\sin x + \cos x = 0$

6.  $3\sec x \tan x - 2 = 0$

### E.4 Compositions with Trig Functions

Find functions  $f$  and  $g$  so that  $k(x) = f(g(x))$ :

1.  $k(x) = \sin^3 x$    2.  $k(x) = \sin x^3$    3.  $k(x) = \cos(2x + 1)$    4.  $k(x) = 2\cos x + 1$

Find functions  $f$ ,  $g$  and  $h$  so that  $k(x) = f(g(h(x)))$ :

5.  $k(x) = \tan^2 \sqrt{x}$

6.  $k(x) = 3\cos x^4 + \frac{5}{\cos x^4}$

### E.5 Graphing Trig Functions

Sketch the graphs of the following functions on the interval  $[0, 2\pi]$ :

1.  $y = 2\cos x$

2.  $y = \cos 2x$

3.  $y = \sin(x - \frac{\pi}{2})$

4.  $y = \sin x - 1$

5.  $y = \tan\left(\frac{x}{2}\right)$

6.  $y = \sec(\pi x)$

7.  $y = -2\sin 3x$

8.  $y = 3\cos \frac{1}{2}(x + \pi)$

## SOLUTIONS AND ANSWERS

### Section A.1

1.  $\frac{-12(-2)^2 - (-3)^3}{3(-2)(-3)^2 - 4(-3)} = \frac{-48 + 27}{-54 + 12} = \frac{-21}{-42} = \frac{1}{2}$
2.  $\frac{7}{4}$
3.  $3(-3)^2(2(-3)+15)^{-\frac{1}{2}} - (-3)^3(2(-3)+15)^{-\frac{3}{2}} = 27(9)^{-\frac{1}{2}} + 27(9)^{-\frac{3}{2}} = 9+1 = 10$   
Recall that  $9^{-\frac{3}{2}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$
4.  $-128$
5.  $((\frac{1}{2})^3 - (-3)^{-2})^{-1} = (\frac{1}{8} - \frac{1}{9})^{-1} = (\frac{1}{72})^{-1} = 72$
6. Recall that  $x^{-1} = \frac{1}{x}$  and  $y^{-2} = \frac{1}{y^2}$ . The resulting expression is undefined because division by zero is not defined.

### Section A.2

1.  $3x^3 + 3x^2 - 2x - 2 - 6 - 2(x^3 - 6x^2 + 12x - 8) = x^3 + 15x^2 - 26x + 8$   
You can expand  $(x - 2)^3$  methodically or use the binomial expansion or Pascal's Triangle that would give:  $x^3 + 3(x)^2(-2) + 3(x)(-2)^2 + (-2)^3$
2.  $-15x^2 - 25x - 18$
3.  $\sqrt{x^4 - 2x^2 + 1 + 4x^2} = \sqrt{x^4 + 2x^2 + 1} = \sqrt{(x^2 + 1)^2} = x^2 + 1$
4.  $4\sqrt{x^2 + 4}$
5. Combine the terms by getting the lowest common denominator to get:  
$$\frac{x+3}{(x+3+h)(x+3)} - \frac{x+3+h}{(x+3+h)(x+3)} = \frac{x+3-(x+3+h)}{(x+3+h)(x+3)} = \frac{-h}{(x+3+h)(x+3)}$$
6.  $-\frac{1}{4x}$

7. Convert to positive exponents and get the lowest common denominator to get:

$$\frac{2}{x+2} - \frac{2x}{(x+2)^2} = \frac{2(x+2)}{(x+2)^2} - \frac{2x}{(x+2)^2} = \frac{2x+4-2x}{(x+2)^2} = \frac{4}{(x+2)^2}$$

8.  $\frac{4x^2}{(2-x)^2}$  or  $\frac{4x^2}{(x-2)^2}$

### **Section B.1**

1.  $2x^5(4x^2 - 1)^4 [3(4x^2 - 1) + 20x^2] = 2x^5(4x^2 - 1)^4(32x^2 - 3)$   
or  $= 2x^5(2x-1)^4(2x+1)^4(32x^2 - 3)$

2.  $2(x^2 + 1)^2(5x + 1)(20x^2 + 3x + 5)$

3.  $(x^2 - 16)(x^2 + 3) = (x - 4)(x + 4)(x^2 + 3)$

4.  $(2x - 3)(x + 5)$

5.  $x^2(2x - 5) - 9(2x - 5) = (2x - 5)(x^2 - 9) = (2x - 5)(x - 3)(x + 3)$

6.  $2(2x + y)(x - y)(x + y)$

### **Section B.2**

1.  $\frac{4(x^2 - 2x - 3)}{(2x + 1)(x - 3)} = \frac{4(x - 3)(x + 1)}{(2x + 1)(x - 3)} = \frac{4(x + 1)}{2x + 1}$

2.  $\frac{3(x + 2)(x^2 + 4)}{x + 6}$

3.  $\frac{\frac{4}{x^2} - \frac{4}{9}}{x - 3} \cdot \frac{9x^2}{9x^2} = \frac{36 - 4x^2}{(x - 3) \cdot 9x^2} = \frac{4(3 - x)(3 + x)}{(x - 3) \cdot 9x^2} = \frac{-4(x + 3)}{9x^2}$

4.  $\frac{-x^2 - 2x - 4}{8x^3}$

5.  $\frac{4}{(x - 1)(x + 1)} + \frac{x}{(x - 3)(x + 1)} = \frac{4(x - 3) + x(x - 1)}{(x - 1)(x + 1)(x - 3)} = \frac{x^2 + 3x - 12}{(x - 1)(x + 1)(x - 3)}$

$$6. \frac{2(x^2 - 2x + 2)}{x(x-1)^2}$$

### **Section B.3**

$$1. 2x^2 - x - 15 = (2x+5)(x-3) = 0 \Rightarrow x = -\frac{5}{2}, x = 3$$

$$2. x = \pm 3$$

$$3. x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)} = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6} = \frac{1 \pm \sqrt{13}}{3}$$

4. no real solutions

$$5. \frac{1}{x} - \frac{2x}{x+3} \Rightarrow x+3 = 2x^2 \Rightarrow 2x^2 - x - 3 = 0 \Rightarrow (2x-3)(x+1) = 0 \Rightarrow x = \frac{3}{2}, -1$$

$$6. x = 1, x = 3$$

$$7. \text{ Cross multiplying gives: } 2(x+1) = \sqrt{x^2 + 2x + 13}, \text{ then square both sides to get:}$$

$$4(x+1)^2 = x^2 + 2x + 13 \Rightarrow 3x^2 + 6x - 9 = 0 \Rightarrow 3(x+3)(x-1) = 0 \Rightarrow x = -3, x = 1$$

Checking answers shows that  $x = -3$  does not work so the only solution is  $x = 1$

$$8. x = \frac{1}{2}, x = 1$$

9. Factoring out the greatest common factor gives:

$$(x+4)^2(2x-1)^{3/2} [3(2x-1) + 5(x+4)] = 0 \Rightarrow (x+4)^2(2x-1)^{3/2}(11x+17) = 0$$

Any of the factors could be zero, so the solutions are:  $x = -4, x = \frac{1}{2}, x = -\frac{17}{11}$

$$10. \text{ Factoring gives: } 2(x+3)^3(x^2-1)^2(5x^2+9x-2) = 0. \text{ Factoring further gives:}$$

$$2(x+3)^3(x-1)^2(x+1)^2(5x-1)(x+2) = 0 \Rightarrow x = -3, x = 1, x = -1, x = -2, x = \frac{1}{5}$$

$$11. x(y+4) = 5-y \Rightarrow xy + y = 5-4x \Rightarrow y(x+1) = 5-4x \Rightarrow y = \frac{5-4x}{x+1}$$

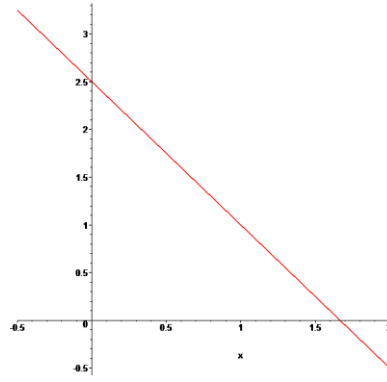
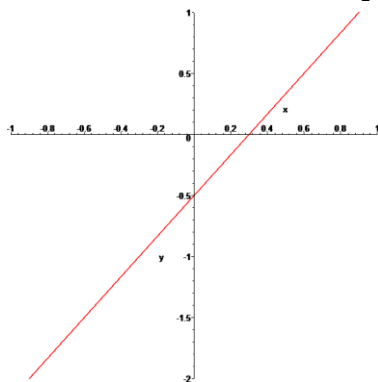
$$12. w = \frac{A-2lh}{2l+2h}$$

### Section B.4

- $x - \frac{5}{2}x + \frac{10}{3} - 1 \geq 0 \Rightarrow 6x - 15x + 20 - 6 \geq 0 \Rightarrow -9x \geq -14 \Rightarrow x \leq \frac{14}{9}$  or  $(-\infty, \frac{14}{9}]$
- $x > -4$  or  $(-4, \infty)$
- $(x-7)(x+2) > 0$  This expression is zero when  $x=7$ ,  $x=-2$ . Using test points between these values and determining the intervals that satisfy the inequality gives the solution:  $(-\infty, -2) \cup (7, \infty)$
- $(-1, 3]$

### Section C.1

- $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$   
 $= \sqrt{(-2 - 6)^2 + (-1 - (-3))^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$
  - midpoint  $= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2 + 6}{2}, \frac{-1 + (-3)}{2} \right) = (2, -2)$
  - slope  $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-3)}{-2 - 6} = -\frac{1}{4}$
  - 4
  - $y - y_1 = m(x - x_1) \Rightarrow y - (-1) = -\frac{1}{4}(x - (-2)) \Rightarrow y = -\frac{1}{4}x - \frac{3}{2}$  or  $x + 4y + 6 = 0$
- $5\sqrt{2}$
  - $(\frac{1}{2}, -\frac{3}{2})$
  - 1
  - 1
  - $y = -x - 1$  or  $x + y + 1 = 0$
- x intercept let  $y = 0 \Rightarrow x = \frac{3}{10}$   
y intercept let  $x = 0 \Rightarrow y = -\frac{1}{2}$
- $x = \frac{5}{3}$   $y = \frac{5}{2}$

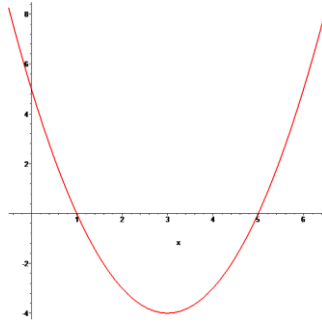


## Section C.2

1. a) For  $x$  intercept, let  $y = 0 \Rightarrow (x-1)(x-5) = 0 \Rightarrow x = 1, x = 5$   
For  $y$  intercept, let  $x = 0 \Rightarrow y = 5$

b)  $y = (x^2 - 6x \quad ) + 5 = (x^2 - 6x + 9) + 5 - 9 = (x-3)^2 - 4$ . Vertex is  $(3, -4)$

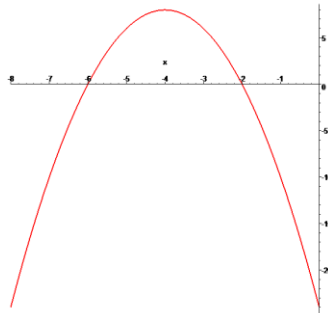
c)



2. a)  $x = -2$   $x = -6$  and  $y = -24$

b)  $(-4, 8)$

c)

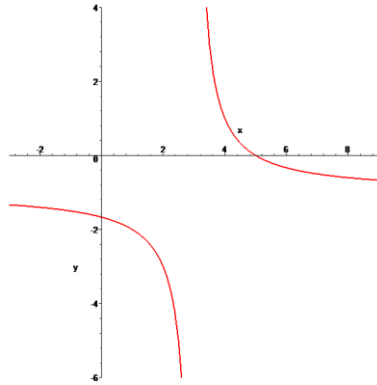


3. a)  $x = 0 \Rightarrow y = \frac{2}{-3} - 1 = -\frac{5}{3}$

$$y = 0 \Rightarrow 0 = \frac{2}{x-3} - 1 \Rightarrow 1 = \frac{2}{x-3} \Rightarrow x-3 = 2 \Rightarrow x = 5$$

- b) Vertical asymptote:  $x = 3$  , Horizontal asymptotes:  $y = -1$

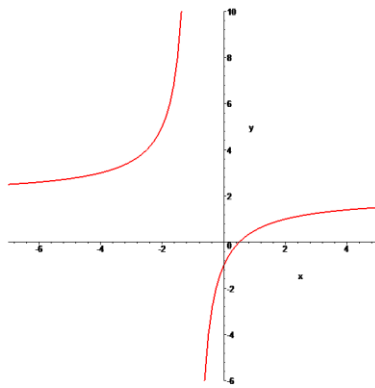
c)



4. a)  $x = \frac{1}{2}$   $y = -1$

b)  $x = -1$   $y = 2$

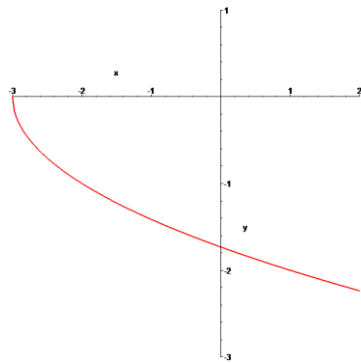
c)



5. a)  $x + 3 \geq 0 \Rightarrow x \geq -3$

b)  $x = -3$   $y = -\sqrt{3}$

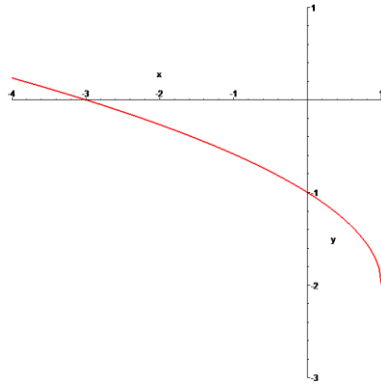
c)



6. a)  $x \leq 1$

b)  $x = -3 \quad y = -1$

c)



### Section D.1

1. Domain:  $2x+1 \geq 0 \Rightarrow x \geq -\frac{1}{2}$

Range: Since  $\sqrt{2x+1}$  is greater than or equal to zero, the range is  $y \geq -2$

2. Domain:  $x \leq \frac{3}{2}$  Range:  $y \leq 5$

3. Domain:  $5x-4 \neq 0 \Rightarrow x \neq \frac{4}{5}$  or  $\{ x \mid x \neq \frac{4}{5}, x \text{ a real number} \}$

Range:  $\frac{3}{5x-4}$  can not be zero so  $y \neq 1$  or  $\{ y \mid y \neq 1, y \text{ a real number} \}$

4. Use long division to rewrite the function as  $f(x) = 3 - \frac{7}{x+2}$ .

Domain:  $x \neq -2$  Range:  $y \neq 3$

5. You can take the cube root of any number, so: Domain and Range are both all real numbers.

6. Domain:  $x \neq \pm 2$  Range:  $y \neq 0$

### Section D.2

1.  $f\left(-\frac{2}{3}\right) = \frac{-\left(-\frac{2}{3}\right)^2 + 2\left(-\frac{2}{3}\right) + 2}{4\left(-\frac{2}{3}\right) - 3} = \frac{\frac{-7}{9}}{\frac{-17}{3}} = -\frac{2}{51}$

2.  $-\frac{10}{3}$

$$3. \quad f(3a) = \sqrt{(3a)^2 + 9} + \frac{1}{\sqrt{(3a)^2 + 9}} = 3\sqrt{a^2 + 1} + \frac{1}{3\sqrt{a^2 + 1}} = \frac{9a^2 + 9 + 1}{3\sqrt{a^2 + 1}} = \frac{9a^2 + 10}{3\sqrt{a^2 + 1}}$$

$$4. \quad f(a-3) = a^2 - 8a + 20$$

$$5. \quad \frac{-2(x+h)^2 + 3(x+h) - 1 - (-2x^2 + 3x - 1)}{h}$$

$$= \frac{-2x^2 - 4xh - 2h^2 + 3x + 3h - 1 + 2x^2 - 3x + 1}{h} = \frac{h(-4x - 2h + 3)}{h} = -4x - 2h + 3$$

$$6. \quad 3x^2 + 3xh + h^2$$

$$7. \quad \frac{\frac{1}{3(x+h)+2} - \frac{1}{3x+2}}{h} = \frac{(3x+2) - (3x+3h+2)}{h(3x+3h+2)(3x+2)} = \frac{-3}{(3x+3h+2)(3x+2)}$$

$$8. \quad \frac{2}{(4-x-h)(4-x)}$$

$$9. \quad \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} = \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \cdot \frac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}}$$

$$= \frac{x+h-1 - (x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$$

$$10. \quad \frac{-2}{\sqrt{1-2x-2h} + \sqrt{1-2x}}$$

### **Section D.3**

$$1. \quad f(x)g(x) = (2x+5)(x^3+1) = 2x^4 + 5x^3 + 2x + 5$$

$$f(g(x)) = 2(x^3+1) + 5 = 2x^3 + 7$$

$$g(f(x)) = (2x+5)^3 + 1 = 8x^3 + 60x^2 + 150x + 126$$

$$2. \quad f(x)g(x) = (2x^2+5)\sqrt{2x-1} ; f(g(x)) = 4x+3, x \geq \frac{1}{2} ; g(f(x)) = \sqrt{4x^2+9}$$

$$3. f(x)g(x) = (x^3 + 8) \cdot \frac{1}{\sqrt[3]{x}} = x^{8/3} + 8x^{-1/3}$$

$$f(g(x)) = \left( \frac{1}{\sqrt[3]{x}} \right)^3 + 8 = \frac{1}{x} + 8 = \frac{1+8x}{x}$$

$$g(f(x)) = \frac{1}{\sqrt[3]{x^3 + 8}} = (x^3 + 8)^{-1/3}$$

$$4. f(x)g(x) = -4, \quad f(g(x)) = \frac{1}{10-9x}, \quad g(f(x)) = \frac{24x-44}{3x-4}$$

$$5. f(x) = x^5, \quad g(x) = 4x^2 - 2x - 1$$

$$6. f(x) = \sqrt{x}, \quad g(x) = 9x^2 + 25$$

$$7. f(x) = x^4 + \frac{1}{x^2}, \quad g(x) = 3x + 2$$

$$8. f(x) = x^{-2/5}, \quad g(x) = x^3 - 27$$

#### **Section D.4**

$$1. A(x) = \frac{82 + 75 + 89 + x}{4} \Rightarrow A(x) = \frac{246 + x}{4}$$

$$2. \text{Radius of the circle is } r = \frac{x}{2\pi} \text{ and the length of a side of the square is } s = \frac{50-x}{4}.$$

$$\text{So, } A(x) = \pi \left( \frac{x}{2\pi} \right)^2 + \left( \frac{50-x}{4} \right)^2 = \frac{x^2}{4\pi} + \frac{x^2 - 100x + 2500}{16}$$

$$3. A(w) = \frac{45}{2}w - w^2$$

$$4. 500 = \pi r^2 h \Rightarrow h = \frac{500}{\pi r^2}$$

$$\text{Therefore: } S(r) = 2\pi r \left( \frac{500}{\pi r^2} \right) + 2\pi r^2 \Rightarrow S(r) = \frac{1000}{r} + 2\pi r^2$$

$$5. D = \sqrt{(x-2)^2 + (y-3)^2} \text{ represents the distance from any point to } (2, 3).$$

$$\text{Therefore: } D(x) = \sqrt{(x-2)^2 + (x^2+1-3)^2} \Rightarrow D(x) = \sqrt{x^4 - 3x^2 - 4x + 8}$$

$$6. A(x) = 2x - \frac{2}{3}x^2$$

## Section E.1

1.  $135^\circ = 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$  radians

2.  $210^\circ$

3.  $1.15$  radians  $= 1.15 \times \frac{180}{\pi} = 65.89^\circ$

4.  $0.06$  radians

5.  $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.577$

6.  $-\frac{\sqrt{3}}{2} = -0.866$

7.  $0.8855$  (Be sure your calculator was in radians)

8.  $-3.236$

9.  $\theta = 60^\circ = \frac{\pi}{3}$  rads

10.  $45^\circ = \frac{\pi}{4}$  rads

11.  $\tan \theta = \frac{1}{2.86} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2.86}\right) = 19.27^\circ = 0.336$  rads

12.  $53.13^\circ = 0.927$  rads

13. Reference angle  $= 60^\circ$ ,  $\cos$  is positive in Quadrant I and Quadrant IV, so the answers between  $0^\circ$  and  $360^\circ$  are  $\theta = 60^\circ$ ,  $\theta = 360^\circ - 60^\circ = 300^\circ$ . Thus, all solutions are:  $\theta = 60^\circ + 360^\circ \cdot k$ ,  $\theta = 300^\circ + 360^\circ \cdot k$  where  $k$  is any integer.

Converting to radians gives:  $\theta = \frac{\pi}{3} + 2\pi k$ ,  $\theta = \frac{5\pi}{3} + 2\pi k$ .

14.  $\theta = \frac{\pi}{4} + 2\pi k$ ,  $\theta = \frac{3\pi}{4} + 2\pi k$

15. Reference angle  $= 30^\circ$ ,  $\tan$  is negative in Quadrant II and Quadrant IV, so the answers between  $0^\circ$  and  $360^\circ$  are  $\theta = 180^\circ - 30^\circ = 150^\circ$ ,  $\theta = 360^\circ - 30^\circ = 330^\circ$ . Thus, all solutions are:  $\theta = 150^\circ + 360^\circ \cdot k$ ,  $\theta = 330^\circ + 360^\circ \cdot k$ , where  $k$  is any integer.

Converting to radians gives:  $\theta = \frac{5\pi}{6} + 2\pi k$ ,  $\theta = \frac{11\pi}{6} + 2\pi k$ . This can be

written more simply as:  $\theta = \frac{5\pi}{6} + \pi k$

16.  $\theta = 3.785 + 2\pi k$ ,  $\theta = 5.64 + 2\pi k$

## Section E.2

1.  $\frac{1 - \cos^2 x}{1 - \sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

2.  $\frac{1}{\tan^2 x} = \cot^2 x$

$$3. \frac{\cos^2 x}{1 - \sin x} = \frac{1 - \sin^2 x}{1 - \sin x} = \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x} = 1 + \sin x$$

$$4. \frac{1 - \sec x}{\sec^2 x - 1} = \frac{-1}{\sec x + 1}$$

$$5. \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = \frac{1 - \cos x + 1 + \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{2}{1 - \cos^2 x} = \frac{2}{\sin^2 x} = 2 \csc^2 x$$

$$6. \frac{\tan x + \sec x - (\tan x - \sec x)}{\tan^2 x - \sec^2 x} = -2 \sec x$$

$$7. \begin{aligned} \sin(x + y) - \cos(x - y) &= \sin x \cos y + \cos x \sin y - (\cos x \cos y + \sin x \sin y) \\ &= \sin x(\cos y - \sin y) - \cos x(\cos y - \sin y) \\ &= (\sin x - \cos x)(\cos y - \sin y) \end{aligned}$$

$$8. \begin{aligned} (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= \sin^2 x - \sin^2 y \quad \text{or alternatively} \quad = \cos^2 y - \cos^2 x \end{aligned}$$

$$9. \sin 6x \quad (\text{No work required!!})$$

$$10. \cos 8x$$

$$11. \frac{2 \sin x \cos x - 2 \sin^2 x}{\cos^2 x - \sin^2 x} = \frac{2 \sin x (\cos x - \sin x)}{(\cos x + \sin x)(\cos x - \sin x)} = \frac{2 \sin x}{\cos x + \sin x}$$

$$12. \frac{2 \sin x \cos x}{1 + (2 \cos^2 x - 1)} = \frac{2 \sin x \cos x}{2 \cos^2 x} = \tan x$$

### **Section E.3**

$$1. \begin{aligned} 2 \sin x \cos x + \sin x = 0 &\Rightarrow \sin x(2 \cos x + 1) = 0 \Rightarrow \sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2} \\ \text{Therefore, the solution set is: } &\{0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}\} \end{aligned}$$

$$2. \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$3. \begin{aligned} 1 - 2 \sin^2 x + \sin x = 0 &\Rightarrow 2 \sin^2 x - \sin x - 1 = 0 \Rightarrow (2 \sin x + 1)(\sin x - 1) = 0 \\ &\Rightarrow \sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = 1 \Rightarrow x \in \left\{ \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2} \right\} \end{aligned}$$

$$4. \left\{ \frac{\pi}{3}, \frac{5\pi}{3}, \pi \right\}$$

$$5. \sin x = -\cos x \Rightarrow \frac{\sin x}{\cos x} = -1 \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4} \text{ or } x = \frac{7\pi}{4}$$

$$6. 3\left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) = 2 \Rightarrow 3\sin x = 2\cos^2 x \Rightarrow 3\sin x = 2(1 - \sin^2 x)$$

$$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow (2\sin x - 1)(\sin x + 2) = 0 \Rightarrow x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6}$$

### Section E.4

$$1. f(x) = x^3 \quad g(x) = \sin x$$

$$2. f(x) = \sin x \quad g(x) = x^3$$

$$3. f(x) = \cos x \quad g(x) = 2x + 1$$

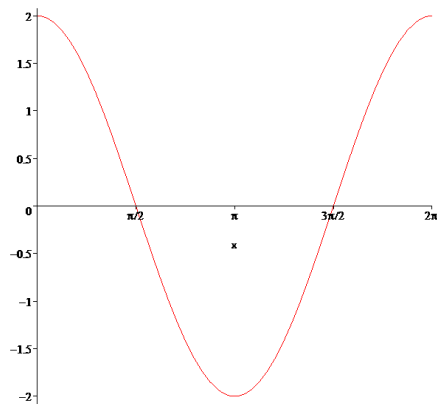
$$4. f(x) = 2x + 1 \quad g(x) = \cos x$$

$$5. f(x) = x^2 \quad g(x) = \tan x \quad h(x) = \sqrt{x}$$

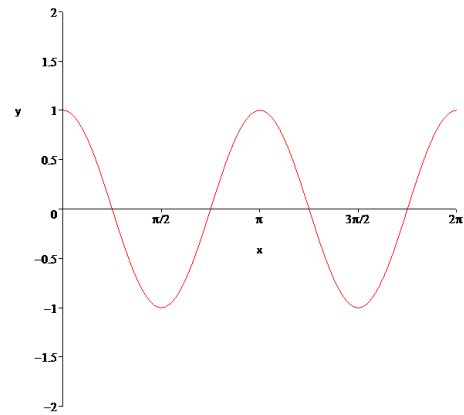
$$6. f(x) = 3x + \frac{5}{x} \quad g(x) = \cos x \quad h(x) = x^4$$

### Section E.5

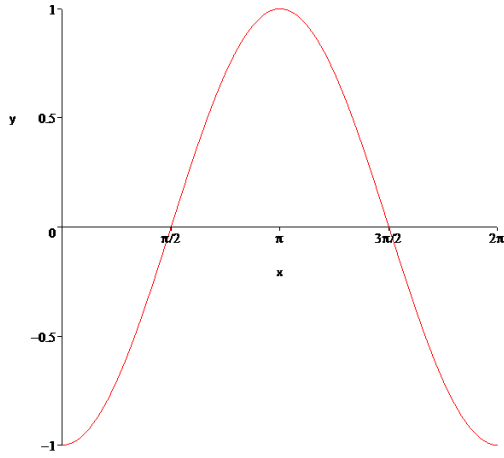
$$1. \text{ Amplitude} = 2 \quad \text{Period} = 2\pi$$



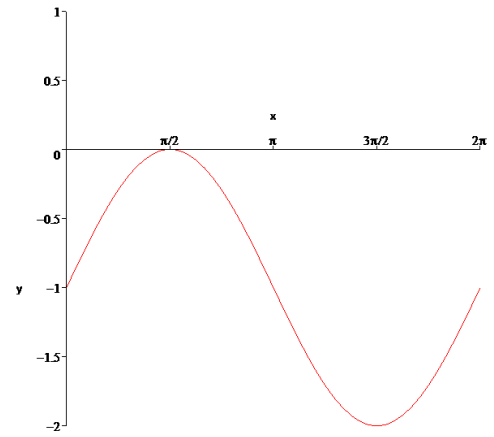
$$2. \text{ Amplitude} = 1 \quad \text{Period} = \frac{2\pi}{2} = \pi$$



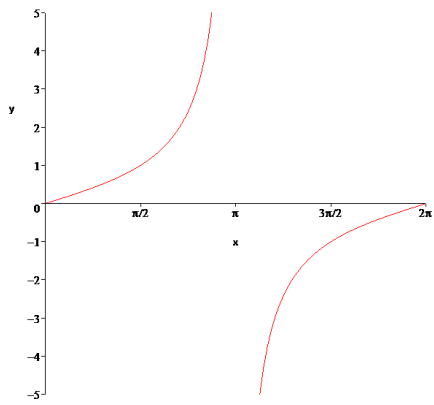
3. Phase shift right  $\frac{\pi}{2}$



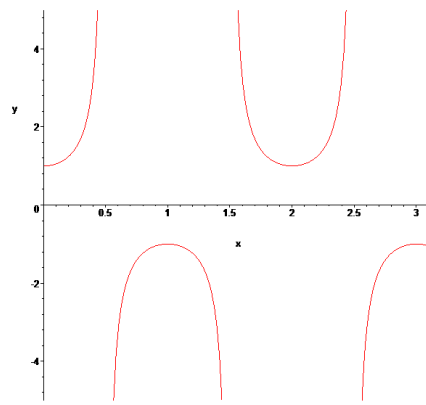
4. Vertical shift down 1



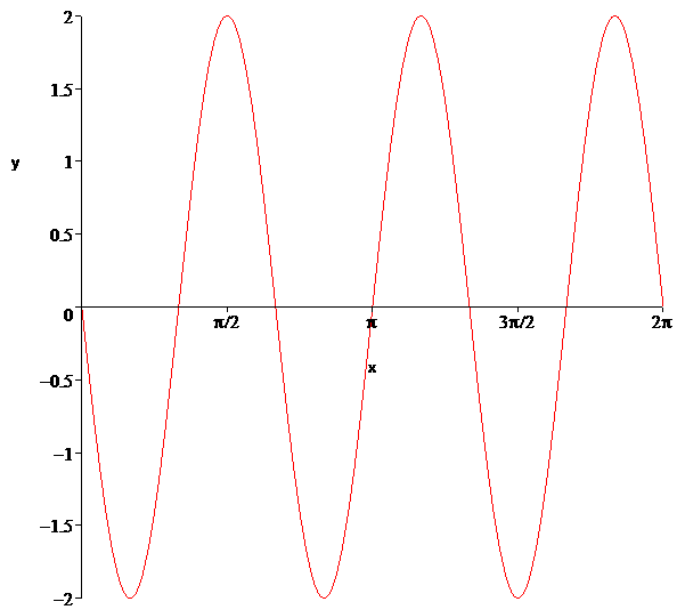
5. Period =  $\frac{\pi}{1/2} = 2\pi$



6. Period =  $\frac{2\pi}{\pi} = 2$



7. Amplitude = 2    Period =  $\frac{2\pi}{3}$     Vertical Flip



8. Amplitude = 3    Period =  $\frac{2\pi}{1/2} = 4\pi$     Phase Shift =  $\pi$  to the left

